Logic Connectives:

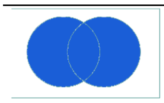
¬ negation	(¬A, pronounced: not A, it is not the fact that A)
∧ conjunction	(A ∧ B, pronounced A and B; only True when A, B are both True)
∨ disjunction, inclusive or	(A ∨ B, pronounced: A or B; only False when A, B are both False)
⊕ exclusive or, XOR	(A ⊕ B; true only if A and B are different, e.g., A is True and B is False)
⇒ implication	(A ⇒ B, pronounced: if A then B, A implies B; only False when the premise is True but the conclusion is False)
⇔ equivalence	(A ⇔ B, pronounced: A if and only if B, A is equivalent to B; only True when A, B are both True or A, B are both False.)

Set notation: {x : x has a property P}

∈ membership	(*a* ∈ A, *a* is a member of A, *a* is an element of A)
∉ non-membership	(*a* ∉ A, *a* is not a member of A, *a* is an element of A)

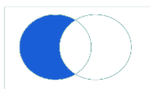
Set A is a **subset** of set B, if, and only if, each element of A is an element of B;

⊂ proper subset	(A ⊂ B, A is a proper subset of B: A is included/contained in B but it is not equal to it)
⊄ not a proper subset	(the opposite of the above, i.e. ¬(A ⊂ B))
⊆ subset (subset equal)	(A ⊆ B, A is a subset of B: A is included/contained in B and it may also be equal to B)
⊈ not a subset	(the opposite of the above, i.e. ¬(A ⊆ B))
⊃ proper superset	(A ⊃ B, A is a proper superset of B: A includes/contains B and it is not equal to it)
⊇ superset	(A ⊇ B, A is a superset of B: A includes/contains B and can be equal to it)



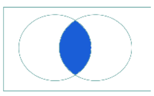
Union: $A \cup B = \{ x : x \in A \text{ or } x \in B\}$

Thus: $x \in A \cup B \Leftrightarrow x \in A \lor x \in B$



Difference: $A \setminus B = \{ x : x \in A \text{ and } x \notin B \}$

Thus: $x \in A \setminus B \Leftrightarrow x \in A \land x \notin B$



Intersection: $A \cap B = \{ x : x \in A \text{ and } x \in B \}$

Thus: $x \in A \cap B \Leftrightarrow x \in A \land x \in B$

Complement: $\bar{A} = \{ x : x \notin A \}$

Thus: $x \in \bar{A} \Leftrightarrow x \notin A$

Powerset P(A) = { X : X ⊆ A }, i.e. it is a set containing all the subsets of A.

$\emptyset = \{ \}$	(empty set; ∅ is a subset of any set)
$\mathbb{B} = \{0, 1\}$	(binary digits, or bits)
$\mathbb{N} = \{0, 1, 2, 3, . . . \}$	(natural numbers)
$\mathbb{Z} = \{. . . , -2, -1, 0, 1, 2, . . . \}$	(integers)
$\mathbb{Q} = \{ \frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0 \}$	(rational numbers)
$\mathbb{R} = \{ x : x \text{ is a real number} \}$	(real numbers)

For intervals, (and) mean that the corresponding elements are not included, [and] means that the corresponding elements are included.

Predicate logic (quantifiers):

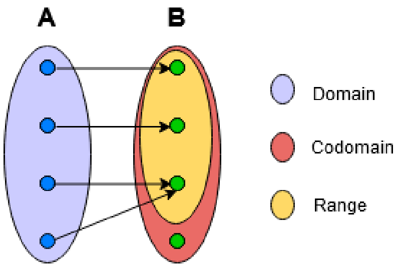
$\forall$ - universal quantifier ($\forall x\ P(x)$: for all x P(x))

$\exists$ - existential quantifier ($\exists x\ P(x)$: there exists x, such that P(x))

$\exists! x\ P(x)$ pronounced "there exists a *unique* x such that P(x)" logical equivalent to: $\exists x\ (P(x) \land \neg \exists y (P(y) \land x \neq y))$

Equivalences: $\neg \forall x\ P(x) \Leftrightarrow \exists x\ \neg P(x)$ $\neg \exists x\ P(x) \Leftrightarrow \forall x\ \neg P(x)$

Functions:



Given a subset $S \subseteq A$ of the domain of f, the ***image*** of S under f is defined by

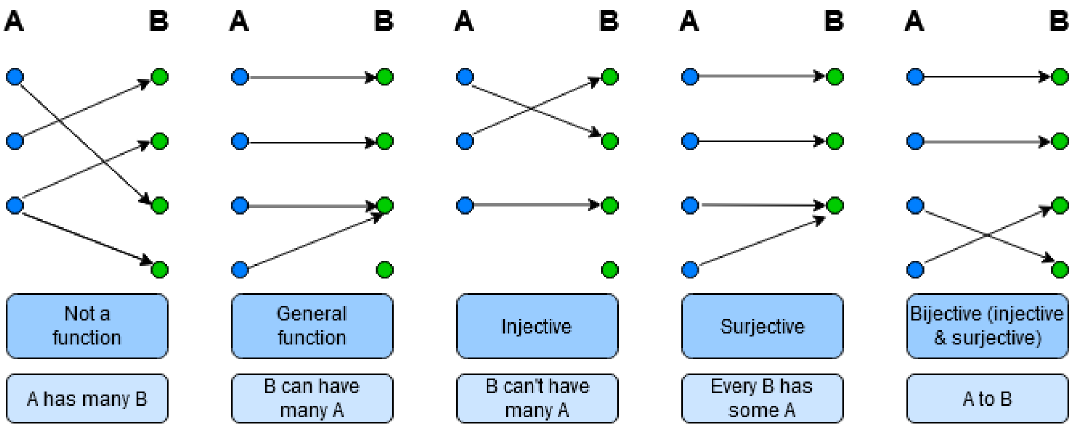
$$f(S) = \{ f(a) : a \in S \}$$

Given a subset $T \subseteq B$ of the codomain of f, the ***preimage*** of T under f is defined by

$$f^{-1}(T) = \{ a \in A : f(a) \in T \}$$

A function $f : A \to B$ can be represented by its ***graph***:

$$graph(f) = \{ (a, b) \in A \times B : b = f(a) \}$$



If $f : A \to B$ is a bijection, then the ***inverse function*** $f^{-1} : B \to A$ is the function that assigns to each $b \in B$ the unique $a \in A$ such that $f(a) = b$:

$$f^{-1}(b) = a \text{ if, and only if, } f(a) = b.$$

Given $f : A \to B$ and $g : B \to C$, the ***composition*** of g and f is the function $g \circ f : A \to C$ defined by

$$(g \circ f)(x) = g(f(x)).$$

Note In order to form composition of g and f, the codomain of f must be the same as the domain of g.

Relations:

- **reflexive** if, and only if, every element of A is related to itself by R.
  That is, $\forall x \in A\ (x\ R\ x)$.
- **irreflexive** if, and only if, no element of A is related to itself.
  That is, $\forall x \in A\ \neg(x\ R\ x)$.
- **symmetric** if, and only if, y is related to x whenever x is related to y.
  That is, $\forall x, y \in A\ ((x\ R\ y) \Rightarrow (y\ R\ x))$.
- **antisymmetric** if, and only if, y is never related to x whenever x is related to y, except possibly for when x = y.
  That is, $\forall x, y \in A\ ((x\ R\ y) \land (y\ R\ x) \Rightarrow x = y)$.

  = is both symmetric and antisymmetric
- **transitive** if, and only if, x is related to z whenever x is related to some y which is related to z.
  That is, $\forall x, y, z \in A\ ((x\ R\ y) \land (y\ R\ z) \Rightarrow (x\ R\ z))$.